4-6 Videos Guide

4-6a

- Notation for triple integrals
  - $\iiint_B f(x,y,z)\,dV = \int_e^f \int_c^d \int_a^b f(x,y,z)\ dxdydz$ for
    $B = \{(x,y,z)|a \le x \le b, c \le y \le d, e \le z \le f\}$
    (a rectangular box—the 2-D trace is R)

  - $\iiint_E f(x,y,z)\,dV = \int_a^b \int_{h_1(x)}^{h_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\ dzdydx$ for
    $E = \{(x.y,z)|a \le x \le b, h_1(x) \le y \le h_2(x), u_1(x,y) \le z \le u_2(x,y)\}$
    (a bounded region in $\mathbb{R}^3$—the 2-D trace is D)

Exercises:

- Evaluate the iterated integral.

$$\int_0^1 \int_0^1 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\ dx\ dy\ dz$$

4-6b

- Evaluate the triple integral.
  - $\iiint_E (x-y)\,dV,$ where E is enclosed by the surfaces $z = x^2 - 1, z = 1 - x^2,$
    $y = 0$, and $y = 2$
  - $\iiint_E z\,dV,$ where E is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x =$
    $0,$
    $y = 3x$, and $z = 0$ in the first octant

4-6c

- Use a triple integral to find the volume of the solid enclosed by the paraboloids
  $y = x^2 + z^2$ and $y = 8 - x^2 - y^2$

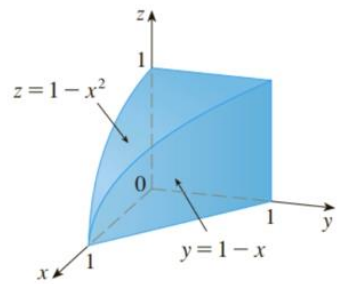
4-6d

- Sketch the solid whose volume is given by the iterated integral.

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\ dz\ dy$$

4-6e

- The figure (on the next page) shows the region of integration for the integral
  $\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\ dy\ dz\ dx$
  Rewrite this integral as an equivalent iterated integral in the five other orders.


z
1
$z = 1 - x^2$
0
1
y
x
1
$y = 1 - x$